How can Germany compete internationally in the field of AI solutions in the future?

Recommendations for action from mathematical associations and professional societies

Germany can and must play a leading role in the next wave of AI innovation by ensuring consistent transfer, interdisciplinary cooperation, and a focus on mathematical excellence in the field of AI. In this joint strategy paper, five relevant associations and professional societies make recommendations for action in the near future and call for this approach.

Germany is currently ranked eighth[(1)] in international comparisons of artificial intelligence (AI), despite being the third-largest economy[(2)]. Nevertheless, Germany is by no means lagging behind and still has a real chance of moving up to the top ranks. Technological breakthroughs can happen very quickly, so a company that is insignificant today could become a leader among the international leaders in just a few months.

To ensure such breakthroughs occur in Germany in the future, business and science must collaborate.

In the context of innovation and technological breakthroughs, this means that the path from research to application, or research transfer, must be better implemented. One area whose potential has not yet been fully exploited is mathematics, even though it forms the foundation of modern AI development. There are enormous, untapped opportunities here that must be seized.

Germany can and must play an active role in shaping the next wave of AI innovation by consistently transferring knowledge, fostering interdisciplinary cooperation, and maintaining a consistent focus on mathematical excellence as a driver in the AI environment.

Action Field I: Transfer Alliance

Much closer cooperation is needed between industry and science, especially in the interaction between mathematics and AI. This must be started quickly. To this end, we propose establishing a transfer alliance. The goal is not to create another competence cluster, but rather to leverage existing structures and networks and connect them in a targeted manner.

A shared win-win vision must be developed primarily by major business partners and academia. It is important to share current problems and necessary contextual knowledge, and to disclose them where possible.

Academia, startups, and medium-sized technology companies can address these challenges and apply this knowledge. In turn, academia contributes solutions based on not yet exploited insights from recent years. Industry selects the approaches with the greatest potential for implementation and market impact from these solutions.

The transfer can then take place through the spin-off of startups, cooperation with medium-sized technology companies, or the targeted recruitment of doctoral students.

Once the alliance is established, we would like to see targeted funding and monitoring from politicians to ensure all partners contribute to the shared vision. This will prevent free-rider effects.

Action Field II: Investment in Basic Research

Sound knowledge is the foundation for forward-looking innovations. Successful AI companies around the world are emerging and flourishing in close proximity to top universities with excellent facilities. To foster this spirit of innovation, we must invest in exceptional research on the mathematical principles of AI, thereby attracting the most talented academics to the German AI ecosystem.

When we identify the potential for practical applications of mathematical inventions, we must immediately begin implementing them. We can strengthen our culture of innovation by supporting mathematical researchers in developing their ideas and actively involving them in the technical realization of their methods.

Contributions from numerical mathematics and mathematical optimization are essential for making neural networks more efficient, smaller, faster, and more reliable. Targeted research in this area has the potential to far exceed the mere optimization and scaling of hardware.

Mathematical research communities are already integrated into various industries, such as finance, insurance, mechanical engineering, transportation planning, and logistics. These industries are based on mathematical theories. Additional AI research will give these and other economic sectors a significant innovation leap by intelligently combining classical and AI methods.

In addition to developing new algorithms, mathematical theory formation is essential for rationally explaining and demystifying AI algorithms. This is an important step toward society's responsible and informed use of AI technology. We aim to apply fundamental knowledge directly to practice, for instance, by training teachers and supporting their continuing education.

Investments in these areas ensure our competitiveness and lay the foundation for a strong, innovative, and future-oriented economy.

Action Field III: Training Future Skilled Workers

In the long term, we face another fundamental challenge that must be addressed today: a shortage of qualified individuals to drive future mathematical and AI-related research. The number of first-year students enrolling in mathematics programs has dramatically decreased in Germany. According to the Federal Ministry for Economic Affairs and Energy, more than 35,000 specialists with in-depth mathematical knowledge of AI applications are needed each quarter, and this number is growing[(3)]. This shortage is further exacerbated by demographic changes and the continuing low status of mathematics in society.

To secure Germany's innovative capacity in the long term, we must significantly strengthen the recruitment and training of talent in mathematics and related disciplines. In addition to targeted support measures and scholarships for students, the social status and visibility of mathematics and its importance for the technological future of our country must be raised.

(1) https://www.laenderdaten.info/groesste-volkswirtschaften.php
(2) https://hai.stanford.edu/ai-index/global-vibrancy-tool
(3) J. Büchel, J. Engler, A. Mertens, "Die Suche nach KI-Fachkräften in Deutschland -- Rekrutierungsstrategien in Stellenanzeigen", Gutachten im Projekt „Entwicklung und Messung der Digitalisierung der Wirtschaft am Standort Deutschland", im Auftrag des Bundesministeriums für Wirtschaft und Klimaschutz, Köln, 2025.

Recommendations for action on the manifesto for mathematics and AI

Action Field I: Accelerating Transfer

Our most important recommendation for action is establishing a transfer alliance for mathematics and AI. Under this umbrella, we will coordinate a number of individual measures.

- Development of a transfer booster program, "Mathematics -> AI."
- Targeted promotion of several innovation networks based on the 'ZIM' model in the field of mathematics and AI.
- Promotion of mathematical contributions to efficient algorithms for AI by the German federal ministry 'BMWE' and the European Union, e.g., within the framework of 'IPCEI'.
- The establishment of an industrial doctorate modeled on the US and UK systems with a one-year internship during the program.
- Targeted support for bridge builders, e.g., through industrial sabbaticals to implement a specific idea in an existing company.
- Risk premiums for established companies that commission AI startups based on 'AGVO' Art. 25, with federal instruments such as those of 'BAFA', 'KfW', and 'SPRIN-D'.

Action Field II: Investment in Basic Research

- Promotion of mathematical AI research by the 'BMFTR'.
- Establishment of a 'DFG' budget for mathematics in AI research.
- Strengthening interdisciplinary research between mathematics, computer science, physics, and engineering in AI research.

Action Field III: Training Future Specialists

We take a holistic view of training future skilled workers, from school to mathematical education at universities.

- The relevance of mathematics for AI must be better known through an information campaign. The target group is primarily school students. We should particularly encourage girls' interest in AI to better tap into the available talent pool.

- A program should be established immediately that provides continuing education opportunities for math teachers to understand the mathematical foundations of AI. Teaching only the "user skills" of AI tools is not sustainable.
- A mathematical understanding of AI must become part of school curricula.
- Data science content should be integrated into university teacher training programs.
- Strengthen mathematical data science degree programs through mathematical oriented AI professorships as part of a federal-state program.

Additions: Examples of technical R&D fields for Mathematics and AI

Results from the June 23, 2025 conference, ‚New Mathematics for AI Innovation', at the Werner von Siemens Center in Berlin.

<u>AI-driven simulation of physical systems</u>

Integrating artificial intelligence (AI), machine learning (ML), and physics-based simulation has the potential to transform how we model, predict, and analyze the properties of complex physical systems. We identify four critical challenges that must be addressed to unlock the full potential of AI-assisted simulation.

- Nonlinear and nonconvex optimization: Improving statistical optimizers and adapting the multiscale properties of loss functions.
- Nonlinear approximation theory: Predicting a priori approximation properties and a posteriori error estimation of neural networks.
- Geometric deep learning and topological data analysis: Graph neural networks, manifold learning, curvature-aware architectures, topological priors, persistent homology, and feature extraction.
- Managing complexity and AI-assisted synthesis of automation solutions and control architectures

<u>AI Approaches in Medical Imaging</u>

Current image reconstruction tasks tend to disproportionately reward methods based solely on empirical performance. To achieve long-term breakthroughs, these methods must ensure the reproducibility and verifiability of their results. Critical aspects such as the convergence properties of iterative methods (e.g., type and speed of convergence), consistency (e.g., exact recovery in a noise-free environment), robustness to noise or model errors, and type of regularization (e.g., explicit or implicit) are often overlooked. Furthermore, the ability to mathematically characterize or prove the robustness and interpretability of a method's internal mechanisms are often not included in the evaluation. This results in the following research needs:

- Development of competitions in medical image recognition that include mathematical guarantees in the evaluation criteria.
- Improvement of the interpretability of AI-supported methods in medical imaging.
- Good dimensioning of the hardware and computing requirements of AI-supported methods, "as small as possible, as large as necessary."

Quantification of Uncertainty for Safe AI

AI algorithms exhibit fascinating capabilities, but they also produce astonishing failures and hallucinations, even for humans. Therefore, detecting uncertainty in the predictions and statements of AI models is of great importance. Uncertainty can be used for online monitoring of autonomously acting agents that follow defensive action protocols in cases of high uncertainty, or for human-in-the-loop procedures when cases with high uncertainty are passed on to human experts for decision-making. Requirements for mathematically oriented AI research include:

- Standardization of uncertainty quantification tools for AI to facilitate their use in economic applications. Development of toolboxes.
- Research into better measurement of relevant uncertainty with regard to the actual risk of application.
- Accurate and scientifically validated methods for determining uncertainty in generative AI, such as detecting hallucinations.
- Secure and mathematically provable differentiation between AI-generated and real data.

Numerical efficiency in training and executing AI models

With the widespread use of AI tools, the energy consumption of AI algorithms has become an important issue, highlighting the importance of efficiency in AI algorithms. Efficiency is also relevant when executing AI models built into mobile applications (edge-capable AI). To develop more efficient AI models, mathematical research is needed for training methods and reducing the size of AI architectures, among other things.

- Improving optimization methods beyond hyperparameter tuning
- Mathematical understanding of the effects of floating-point arithmetic of varying accuracy on training processes
- Adaptive methods in inference through gating — which model solves which task efficiently?
- Method collections and decision aids for training AI algorithms in practice

The following professional societies and associations have signed this strategy paper:

- German Mathematical Society (DMV)
- Society for Applied Mathematics and Mechanics (GAMM)
- Society for Inverse Problems (GIP)
- Society for Operations Research (GOR)
- German AI Association
- Committee for Mathematical Modeling, Simulation, and Optimization (KOMSO)

Berlin, December 2025

On the Internet at https://www.mathematik.de/dmv/dmv-stellungnahme

List of Abbreviations

- AGVO – General Block Exemption Regulation (https://eur-lex.europa.eu/EN/legal-content/summary/general-block-exemption-regulation.html)
- BAFA – Federal Office for Economic Affairs and Export Control (https://www.bafa.de/EN)
- BMFTR – Federal Ministry of Research, Technology and Space (https://www.bmftr.bund.de/EN/)
- BMWE – Federal Ministry for Economic Affairs and Energy (https://www.bundeswirtschaftsministerium.de/Navigation/EN/)
- DFG – German Research Foundation (https://www.dfg.de/en)
- KfW – Staatliche Förderbank KFW (https://www.kfw.de/kfw.de-2.html)
- IPECI – Important Projects of Common European Interest (https://competition-policy.ec.europa.eu/state-aid/ipcei_en)
- SPRIN-D – Federal Agency for Breakthrough Innovations (https://sprind.org/en)
- ZIM - Central Innovation Programme for small and medium-sized enterprises (https://www.zim.de)